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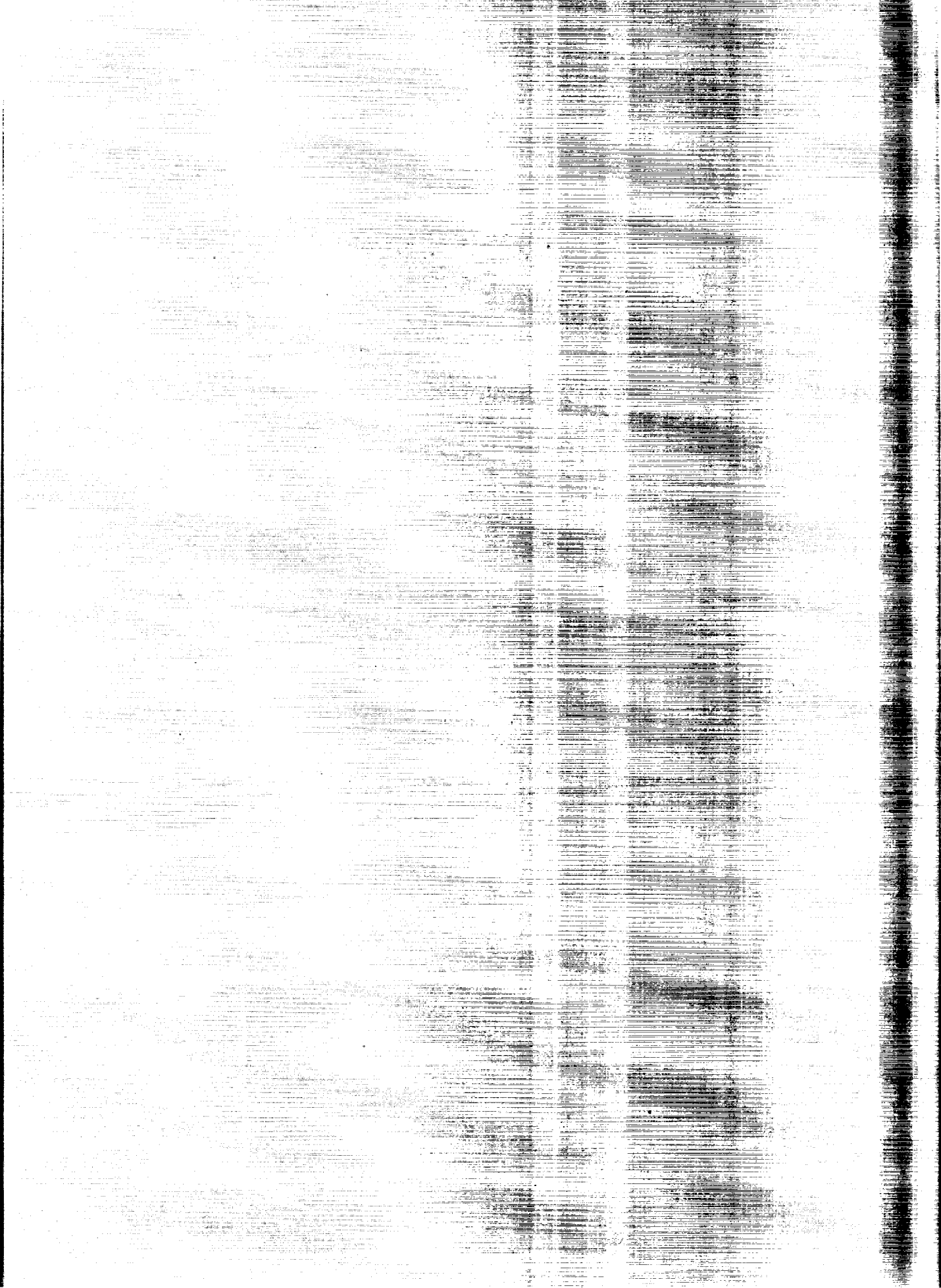
EXPERIMENTAL INVESTIGATION OF IMPACT IN LANDING ON WATER

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EXPERIMENTAL INVESTIGATION OF IMPACT IN LANDING ON WATER\*

By R. L. Kreps

The extent of agreement of the theoretical impact computations with the actual phenomenon has not as yet been fully clarified. There is on the one hand a certain imperfection in the theory (simplifying assumptions made) and on the other an insufficiency in the experimental data available. The object of our present paper is to show how far test results agree with the available approximate computation methods, to investigate in greater detail the physical nature of impact on water, and to perfect the experimental method of studying the phenomenon.

It is shown that the vertical immersion of a freely falling body of a given shape is determined by the nondimensional parameters  $\nu = \frac{gB}{V_0^2}$  and  $\mu = \frac{m}{M}$  (where  $B$  is

a characteristic dimension;  $g$  the acceleration of gravity;  $V_0$  the velocity of the body at impact;  $m$ , the associated mass;  $M$ , the mass of the falling body). For a sharply defined impact having the weight of the body equal to the aerodynamic force, as is the case in the landing of seaplanes, the impact phenomenon may be determined by only the nondimensional parameter  $\mu$ .

For the analysis of the obtained experimental results the fundamental impact computation formulas are given. The experimental procedure is described and the accuracy attained indicated. The tests were conducted on a number of different shapes: a disk, a disk with aperture, a wedge or V shape, and a wedge with slots. The results of all these tests showed a deviation of the test curves of the ratio of velocities before and after impact from the corresponding theoretical results computed by the formulas of Wagner. The reasons for these deviations is explained by the failure to take into account in the equation of motion of the body during impact of the weight of the body and the resistance of the water.

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\*Report No. 438, of the Central Aero-Hydrodynamical Institute, Moscow, 1939.

The tests on the V shapes indicated a considerable decrease in the discontinuous velocity change during impact with increasing wedge angle.

From the theoretical investigations it follows that the presence of even a small slot on the surface of the body sharply reduces the magnitude of the associated mass on impact. The dropping tests conducted on a slotted V shape and on a disk with aperture did not show for these bodies any corresponding decrease in the impact velocity change. The experimental determination, however, of the associated mass of the various bodies with openings gave, in complete agreement with theory, a considerable decrease in the associated mass of these bodies as compared with similar bodies without openings. An analysis of these facts is given which leads to the conclusions that in a number of cases the discontinuous velocity change on impact is not with sufficient accuracy determined by the usually accepted formula  $\frac{V}{V_0} = \frac{1}{1 + \mu}$ . It may be assumed that by taking into account the resistance of the water the agreement of the experimental results with the theory would be considerably improved.

## INTRODUCTION

The problem of the vertical landing of a rigid body on a heavy incompressible liquid is of interest from the point of view of a number of practical applications. The mathematical solution of this problem (on the assumption of the nonturbulent character of the resulting motion) reduces to that of finding a velocity potential  $\phi_1$  which satisfies the Laplace equation  $\Delta\phi_1 = 0$  for the following boundary conditions:

On the free surface:

$$-\rho \frac{\partial \phi_1}{\partial t} - \frac{1}{2} \rho w_1^2 - \gamma y_1 = D \quad (1)$$

where

$\rho$  is the density

$w_1$  the velocity of the liquid on the free surface

Along the wetted area of the body:

$$\frac{\partial \varphi}{\partial n_1} = V_{n_1} \quad (2)$$

where  $V_{n_1}$  is the projection on the inner normal to the surface of the body of the velocity of immersion.

At infinity:

$$(w_1)_\infty = 0 \quad (3)$$

To obtain the law of motion, it is necessary to integrate the equation (the body is assumed to have a translational downward motion)

$$M_1 \frac{dV_1}{dt} = M_1 g - \int_S (p_1 - p_0) dS_x \quad (4)$$

where

$M_1$  is the mass of the immersed body

$V_1$  velocity of the immersed body

$p_1 - p_0$  the excess pressure on the surface of the body

$dS_x$  projection of an element of the wetted surface on horizontal plane

and to find all the magnitudes of interest (forces, velocities, pressures, etc.). Because of the existence of a number of mathematical difficulties, however, this problem has not received an exact solution. These difficulties are due chiefly to the unsteady character of the motion of the fluid when the body is immersed, the nonlinearity of the conditions on the free surface, and also the presence of flow phenomena and spray formation leading to discontinuous motions. The impossibility of solution of the complete problem of the immersion of a body makes it necessary to consider individual approximate cases.

The simplest scheme to consider is that of the impact of a rigid body on the surface of water (reference 2). Impact represents, as is known, a limiting case of motion with instantaneous change in velocity (velocity discon-

tinuity). Thus from the physical point of view the substituting of the immersion process by impact and hence by a velocity discontinuity in place of a velocity change leads from the mathematical point of view, to a simplification of the conditions on the free surface and therefore to a simplification in the determination of the velocity potential  $\varphi_1$ . Thus, since the pressure on the free surface is finite, the impulsive pressure ( $p_{t_1}$ ) along this surface is equal to zero

$$p_{t_1} = \lim_{\Delta t} \int_0^{\Delta t} p_1 dt = -\rho \varphi_1 = 0 \quad (5)$$

and for the determination of the velocity potential  $\varphi_1$  in the limiting case of impact the following boundary conditions obtain:

On the free surface:  $\varphi_1 = 0$

On the surface of the body:  $\frac{\partial \varphi_1}{\partial n_1} = V_{n_1}$

In the solution of the problem of impact of a floating body as also in general in the mathematical investigation of the phenomenon of impact itself, the infinitely large forces are naturally replaced by impulses (references 3 and 4).

From considerations of the impact phenomenon of a floating body an approximate solution of the problem of a continuous nonstationary motion during immersion of a body in a liquid was first obtained by Wagner (references 5 and 6). As a basis of this solution it is assumed that for very sudden immersion of the body the motion of the liquid at each given instant of time may be considered similar to that arising from the impact of the corresponding floating body. Wagner, moreover, takes into consideration the fact that in computing the forces acting on the immersed body it is necessary to take into account the increase in the wetted area of the body due to the motion of the fluid. This is done by the introduction of a function giving the ratio of the velocity of immersion to the rate of increase of the wetted area. The value of this function depends essentially on the shape of the immersed body.

For the solution of the problem of immersion of a wedge-shape body with small keel angle (a shape which is of

interest for seaplanes) Wagner assumes that at each given instant of time the flow arising from the immersion of the body will be identical with the flow due to the impact of a flat plate of width equal to the wetted width of the body under consideration.

Note that all present methods of computing the landing of seaplanes on water are entirely based on the ideas brought out in the theory of Wagner (reference 7). The correspondence of the theoretical computations with the real phenomena of impact are not as yet entirely clear, since on the one hand there is a certain imperfection in the theory (simplifying assumptions made) and on the other hand the experimental data available at the present time do not permit of drawing the required final conclusions. At the same time the substitution of theoretical impact computations for experimental investigations appears very attractive. In particular, it is essentially possible to compute the impact of the schematized contours of the step portions of floats. Generally, the results of towing tests on investigated sections are made the basis of the choice of contours. The results of these tests on the hydrodynamic qualities of the section give no indication, however, of its "impact" properties. The possibility of supplementing the towing tests by an analysis of the sections from the impact point of view would facilitate the problem of a rational choice of the shapes of hulls and floats.

As has already been pointed out, together with the theoretical investigations of the problem of the immersion of rigid bodies in an incompressible liquid, experimental work has been carried out in determining the forces acting on the impact. A large part of all these impact tests is devoted to full scale investigation of the airplane, in particular, to the study of the loads, deformations and pressures (references 8 to 17).

These tests have been occasioned by the urgent requirements of practice, but provide no estimate of the degree of accuracy of the theory because of the presence of a large number of factors that complicate the phenomenon (elasticity of the structure, three-dimensional character of the phenomenon, wave formation on the water surface, landing conditions, etc.). Special laboratory investigations suitable for comparison with the theory are few in number. Of these may be mentioned the work of A. Povitsky (reference 18) at CAHI and of Watanabe (reference 19) in Japan.

A very interesting picture may be obtained of the fall of a body on water and the resulting motion of the liquid under laboratory conditions with the aid of a high-speed movie camera. At the same time, a quantitative presentation, together with the purely qualitative picture of the phenomenon, is obtained of the interval of immersion up to the edges of the body. The high-speed picture obtained in 1937 at CAHI (reference 18) of the dropping on water of various bodies (wedge, disk) showed that the length of the immersion interval of these bodies is very small and, in certain cases, is given in thousandths of a second (the duration of the impact depends on the weight and the initial velocity as well as on the shape of the body). The data thus obtained indicate that it is fundamentally permissible to regard the immersion of the bodies as an almost instantaneous process. With the aid of the photographs here shown (figs. 1 and 2) the successive steps in the immersion of these bodies can be followed.

The object of the present paper is to clarify the correspondence of the test results with those of the available computational methods and also to perfect the experimental methods for studying the impact of a body on water.

#### EQUATIONS OF IMPACT ON WATER AND BASIC COMPUTATIONAL FORMULAS

The exact equation (in nondimensional form) free fall of a rigid body on the surface of water will be given. The given characteristic magnitudes are the velocity of fall of the body at the initial instant of immersion  $V_0$ , the width of the body  $B$ , and the density of the liquid  $\rho$ . The nondimensional magnitudes will be introduced with the aid of the formulas (reference 20)

$$\left. \begin{aligned} x_1 &= Bx; \quad y_1 = By; \quad t = \frac{B}{V_0} \tau; \quad \varphi_1 = V_0 B \varphi \\ M_1 &= \rho B^3 M; \quad p_1 - p_0 = \rho V_0^2 p; \quad V_1 = V_0 V \end{aligned} \right\} \quad (6)$$

The boundary conditions (1) and (2) assume the form

$$\frac{1}{2} w^2 + \frac{\partial \varphi}{\partial \tau} - v_y = 0 \quad (7)$$



$$\frac{\partial \phi}{\partial n} = V_n \quad (8)$$

where  $v$ , equal to  $gB/V_0^2$ , is the reciprocal of the Froude number.

The equation of motion of the immersed body in non-dimensional form becomes

$$M \frac{dV}{d\tau} = Mv - P \quad (9)$$

where

$$P = \frac{1}{\rho B V_0^2} \int_S (p_1 - p_0) dS_x$$

The nondimensional magnitude  $P$  depends on  $v = gB/V_0^2$  and  $V(\tau)$  - that is,

$$P = P[V(\tau), v]$$

Thus

$$M \frac{dV}{dt} = Mv - P[V(\tau), v] \quad (10)$$

From equation (10) and conditions (7) and (8) defining the motion of the liquid, it follows that in the fall of a rigid body on water the motion of the water and of the body is characterized by the two constants:

$$v = \frac{gB}{V_0^2} \quad (\text{reciprocal of Froude's number}) \quad \text{and} \quad M = \frac{M_1}{\rho B^3}$$

(mass coefficient). Hence the immersion of geometrically similar bodies will be dynamically similar if the condition of constancy of the numbers  $v$  and  $M$  is satisfied. For the impact phenomenon, which is characterized by a rapid change in velocity, the accelerations of the body and of the water particles will be very large. In this case it is permissible to neglect also the weight, and for dynamic similarity it is sufficient to satisfy the constancy of the mass coefficient.

In the landing of a seaplane on water the weight of the latter balances the aerodynamic lift force of the wings, but the weight of the water does not essentially affect

the disturbed motion of the water because of the small depth of immersion and the displacement of the liquid particles. For this reason, it is permissible in the given case to consider the immersion phenomenon as determined only by the mass coefficient. In those cases where the effect of the number  $\nu$  is negligibly small, the following conclusion is justified: The expressions of all non-dimensional magnitudes associated with impact on water as functions of the nondimensional time  $\tau$  may contain as parameter only the mass coefficient. In particular, all asymptotic values of the nondimensional magnitudes depend only on the mass coefficient  $M$ , and not on the Froude number, and therefore also on the initial sinking velocity  $V_0$ . This theoretical conclusion is confirmed by tests the results of which will be presented in what follows.

In the application to a very sudden nonstationary immersion the reaction of the water at each time instant may be assumed as given by (reference 5)

$$\frac{d}{dt} (mV) \quad (11)$$

where

$V$  velocity of body

$m$  associated mass

(The magnitudes are here dimensional.)

If the associated mass during the immersion of the body is at each instant assumed equal to the associated mass for the wetted portion of the body, then considering only the reaction of the water, the equation may be written

$$M \frac{dV}{dt} = - \frac{d}{dt} (mV) \quad (12)$$

whence

$$V = \frac{V_0}{1 + \mu} \quad (13)$$

where

$V_0$  initial velocity

$\mu = \frac{m}{M}$  a nondimensional coefficient of which the value for a characteristic immersion may be taken as a basic parameter instead of the mass coefficient

The dynamic character of the phenomenon is partly taken into account considering the true wetted area which is increased by the motion of the displaced liquid.

For the case of a V-shape body of very small V angle the associated mass is taken equal to the associated mass of a flat plate (reference 3)

$$m = \frac{\pi}{2} \rho c^2 l \quad (14)$$

This expression may, however, be made somewhat more accurate. Since under real conditions a body of definite V angle and finite length-to-width ratio is being dealt with, it is necessary to introduce a correction for the V angle ( $\xi_1$ ) and finite aspect ratio ( $\xi_2$ ). The first of these is based on the exact solution of the problem of impact of a V shape (reference 2):

$$\xi_1(\beta) = \frac{2 \tan \beta}{\pi} \left[ \frac{\Gamma\left(\frac{3}{2} - \frac{\beta}{\pi}\right) \Gamma\left(\frac{\beta}{\pi}\right)}{\Gamma\left(\frac{1}{2} + \frac{\beta}{\pi}\right) \Gamma\left(1 - \frac{\beta}{\pi}\right)} - 1 \right] \quad (15)$$

where

$\Gamma(x)$  Euler function.

For small V angles this correction may be written in a more simple form:

$$\xi_1 = 1 - \frac{\beta}{\pi} \quad (16)$$

The correction for finite aspect ratio  $\xi_2$  obtained by Pabst as a result of analysis of experimental data obtained in tests on a flat plate is (reference 20)

$$\xi_2(\lambda) = \frac{\lambda}{\sqrt{1 + \lambda^2}} \left( 1 - 0.425 \frac{\lambda}{1 + \lambda^2} \right) \quad (17)$$

For small values of  $\lambda = \frac{2c}{l} \ll 0.7$ , the correction becomes

$$\xi_2 = 1 - \frac{1}{2} \lambda \quad (18)$$

The final formula for the associated mass of a V shape is thus

$$m = \frac{\pi}{8} \rho B^2 \xi_1 \xi_2 \quad (19)$$

where  $B = 2c$ , the entire width of the body.

It may be shown that for two dimensional symmetrical bodies of slightly curved keel the derivative of the immersed depth with respect to the wetted half width is connected with the shape of the body by the integral relation

$$y(x) = \int_0^x \frac{u(c)dc}{\sqrt{1 - \frac{x^2}{c^2}}} \quad (20)$$

where

$$u(c) = \frac{dh}{dc}$$

$c$  wetted half-width

$h$  depth of immersion

$y(x)$  equation of cross-section of body

For  $y(x) = x \tan \beta$  (straight keeled bottom)  $u(c) = \frac{2}{\pi} \tan \beta = \text{constant}$  - that is, because of the motion of the water the wetted surface of the wedge becomes  $\frac{\pi}{2}$  times as large. To the increased wetted width of the wedge there corresponds the depth  $h_2 = \frac{\pi}{2} h$  (fig. 3). For the interval of immersion in impact

$$t = \frac{1}{V_0} \int_0^t (1 + \mu) dt = \frac{1}{V_0} \int_0^c (1 + \mu) u dc \quad (21)$$

For a wedge

$$t = \frac{h}{V_0} \left( 1 + \frac{1}{3} \mu \right) \quad (22)$$

If it is assumed that in the immersion of a wedge up to its geometric boundaries, it is also acted upon by the resistance force of the water  $kV^2$

where

$$k = \frac{C_D}{2} \rho S$$

$C_D$  resistance coefficient

$\rho$  density,  $\frac{\text{kg} \cdot \text{sec}^2}{\text{m}^4}$

$S$  wetted area

the equation of motion assumes the form

$$\frac{d}{dt} [(1 + \mu)V] = - \frac{k}{M} V^2 \quad (23)$$

and the velocity

$$V = \frac{V_0}{(1 + \mu)^{1+\delta}} \quad (24)$$

where

$$\delta = \frac{2 C_D \tan \beta}{\pi^2}$$

Thus, in taking into account the resistance  $\frac{C_D}{2} \rho S V^2$ , the result is formula (24) instead of formula (13). If it is assumed that  $C_D = 1.28$  for a body with small  $V$  angle (this value corresponds to data of aerodynamic tests); then  $\delta = 0.815 \tan \beta$ .

## IMPACT TEST PROCEDURE

## Measurement Accuracy

The impact tests were based on the procedure worked out earlier at the CAHI (reference 21), which consists essentially of finding the discontinuous velocity change during impact. According to formula (13) for the ratio of the velocities before and after the impact  $V/V_0$

$$\frac{V}{V_0} = \frac{1}{1 + \mu}$$

where

$$\mu = \frac{m}{M}$$

The direct test procedure with our modifications may be described as follows:

On the object tested (for example, a wedge (fig. 4)) a vertical support is mounted to which in turn are attached at certain intervals two electric lamps fed by a 24-volt storage battery. The test object is suspended by a string or thin rope over pulleys to a beam located on the surface of the water. An additional rope is attached, serving to raise and fix the body at the given height. In addition to checking the vertical position of the body above the horizontal water surface, a careful control is required of the horizontal position of the body as a whole. This check is made with the aid of two levels situated on the upper surface of the body.

In carrying out the tests a 2- by 3- by 1-meter tank was employed the superstructure of which permitted raising the tested object to a height of 2.25 meters. The body was released by cutting the suspension ropes. The motion of the falling body - that is, of the light sources - was filmed by a photographic camera having in front of its objective a slotted disk rotated by an electric motor. The rotating disk gave a series of short strokes on the film for the motion of the light source instead of a continuous line. (See fig. 5.) From the lengths of these strokes and the distance between them the falling velocity of the body for various instants of time could be easily determined as follows:

Knowing the speed of the rotating disk (the rotational

speed of the motor is fixed with the aid of an attached tachometer) and the number of slots in the disk, the time interval can be found between the recording of two successive strokes. The paths traversed by the light sources can also be easily determined by measurement on the film of the distance between the strokes with a subsequent reduction of these magnitudes to the true values of the distances fallen by the body. The lengths on the film are measured with the aid of a special reading comparator giving a magnification of 10 times. Since the negative has the traces of two lamplight sources, by finding the distance between the latter and knowing the true position of the lamps on the body, the measure is obtained for the full-scale computation of the data obtained on the film. From the photographs obtained, the distances and falling velocities of the body can be determined.

$$V = \frac{La}{l \Delta t} \text{ meters per second} \quad (25)$$

where

$l$  distance between lamps on photograph

$L$  actual distance between lamps

$a$  distance traversed by light source in a given time interval (on the film)

$\Delta t$  time interval reading

$V$  true velocity of motion of body

Each object with definite load is tested for various dropping heights — that is, for various initial impact velocities  $V_0$ . For computing the ratio of velocities before and after impact  $V/V_0$ , a diagram is constructed of the falling velocities of the body for each interval of time  $\Delta t$ . Because of the smallness of this interval there are usually constructed not the instantaneous velocities as a function of the time but the distances traversed by the body in these time intervals and the velocity ratio  $V/V_0$  is computed as the distance ratio  $a/a_0$  before and after impact for the same time interval  $\Delta t$ . From the analysis of one film record, a series of increasing values of  $a$  are obtained corresponding to the increase in the velocity of the body falling in air; then a few values of  $a$  corresponding to certain mean values of the

velocity at the instant of impact; and finally a series of values of  $a$  corresponding to the velocity of immersion with small acceleration (fig. 6).

To determine the velocities directly before and after impact, the procedure is as follows. Two straight lines are drawn between the points corresponding to the motion of the body in the air and the points corresponding to the slow immersion in water and through the intermediate points a line is drawn intersecting the first two lines. The points of intersection thus obtained determine (with a certain degree of approximation) the required velocities  $V$  and  $V_0$ . It is quite clear that the smaller the cut-off time interval the more accurately it is possible to establish the beginning and the end of the impact, and hence the velocity ratio  $V/V_0$ .

Computing the duration of the impact by the formula

$$t = \frac{2h_2}{\pi V_0} \left( 1 + \frac{gm}{3G} \right) \quad (26)$$

(where  $G$  is the weight of the wedge in kg) for the wedge with angle  $50^\circ 40'$  for values of  $\mu$  from 4.66 to 2.33 and initial velocities  $V_0$  from 3 to 5 meters per second, the duration of the impact was obtained as 0.003 to 0.006 second. As may be seen from formula (26) the time  $t$  decreases with increase in the weight  $G$  and the initial velocity  $V_0$ . An attempt was made therefore to obtain records of the distance traversed by the body in time intervals considerably less than those indicated. In our experiments the cut-off time interval was equal to 0.00166 second (in some tests it extended to 0.006 sec). The limiting size of the time interval is given by the distance that it is possible to separate the small strokes corresponding to the motion of the lamps on the photographic film. To increase the scale of the strokes and the intervals between them, it was necessary to use photographic apparatus of dimensions 40 by 50 centimeters. A further decrease in the cut-off time interval requires the development of special apparatus for recording the motion of the falling body.

In concluding the description of the experimental procedure for the impact experiments, the question of the accuracy in the velocity determination will be considered. The errors are those involved in filming the falling body



and reading the records. The first depends essentially on the uniformity of rotation of the camera disk: in other words, on the equality of the time intervals between the passage of two neighboring slits of the disk in front of the objective. The calibration of the film record indicates that the error due to the nonuniformity of the disk rotation does not exceed 0.5 percent.

The accuracy in the computation of the velocity change may in turn depend on two factors: namely, on the accuracy of the measurement of the lengths of the strokes on the film and on the accuracy of drawing the lines on the velocity diagram, especially the intersecting straight line. The accuracy of measurement of the lengths depends on the accuracy in reading the comparator apparatus and on the personal errors of the experimenter.

The accuracy possible with the comparator used was equal to 0.001 mm. The personal errors are due mainly to the difficulty in determining the start of the stroke on account of the blurring of the edges.

The best method for estimating the subjective errors is a second reading of the same film record. According to check readings the deviation in magnitude of a measured length does not exceed 3 percent, but usually it fluctuates within the limits of 0.5 to 1 percent.

The errors in the determination of  $V_0$  and  $V$  will now be considered. The maximum error in the determination of the instant of start of impact (i.e., the point of intersection of the upper straight line with the secant) cannot be greater than the time interval  $\Delta t$ . Since up to the instant of impact, the motion, roughly speaking, is that of a freely falling body ( $V_1 = V^* + gt$ , where  $V^*$  is the initial velocity), if the mean value of the interval  $\Delta t = 0.003$  second, it is found that for the time interval  $1/2 \Delta t = 0.0015$  second the body develops the additional velocity  $\Delta V = 9.81 \times 0.0015 \approx 0.015$  meter per second. This absolute error does not depend on the value of the initial velocity. If it is remembered that in the tests the velocity before impact  $V_0$  varied within the range of 0.8 to 8 meters per second, the values of the maximum relative error is found to lie within the limits 2 to 0.2 percent.

This computation of the errors again confirms what has been stated above — namely, that with decrease in the

cut-off interval there is an increase in the accuracy of determination of the velocity change in impact. The error in computing the velocity after impact  $V$  — that is, the location of the points of intersection with the lower straight line — is very small (of the order of 0.5 percent) since at the start of the immersion with small acceleration the velocity of the body changes very slowly (the straight line is very slightly inclined from the horizontal).

Resuming the above discussion with regard to the accuracy in the determination of the velocity ratio  $V/V_0$ , it may be stated that the method described assured an accuracy within the limits of 1.5 to 4.0 percent.

#### TESTS WITH THE DISK

One of the objects selected in the investigation of the impact phenomena for a body dropped on water was a disk, the latter being a shape for which an accurate theoretically computed expression for the associated mass ( $m = 4/3 \times \rho r^3$ , where  $r$  is the disk radius (reference 3)) is available. The disk was of aluminum and had a diameter  $D = 0.5$  meter and thickness  $h = 7$  millimeters. The tests were conducted for two weights of the disk (different loadings per unit area)  $G = 8.1$  kilograms ( $\mu = 1.33$ ) and  $G = 15.66$  kilograms ( $\mu = 2.56$ ) within the range of initial velocities  $V_0$  from 1.25 to 7.25 meters per second.

From the test results it appeared that the test curves obtained for the velocity ratio  $V/V_0$  for the disk do not coincide with the corresponding curves computed by the formula

$$\frac{V}{V_0} = \frac{1}{1 + \mu}$$

the test values of  $V/V_0$ , up to a certain value of  $V_0$  being greater than the theoretical (fig. 7).

For the disk under consideration for  $G = 8.1$  kilograms, the computed velocity ratio  $\frac{V}{V_0} = 0.281$  and for  $G = 15.66$  kilograms  $\frac{V}{V_0} = 0.429$ : The experimental values

for the first weight change from  $\frac{V}{V_0} = 0.345$  to  $\frac{V}{V_0} = 0.24$  and for the second weight change from  $\frac{V}{V_0} = 0.51$  to  $\frac{V}{V_0} = 0.34$ .

As may be seen from the above figures, the difference between experiment and computation shows up particularly sharply for the larger weight of the disk. Thus, judging by the test results, it may be concluded (in correspondence with theoretical investigations (see section 2)) that the velocity ratio  $V/V_0$  for small initial velocity and large weight depends on the initial velocity of impact  $V_0$  as well as on the mass of the body. According to the theory of Wagner, however, the velocity ratio  $V/V_0$  does not depend on the initial velocity.

In the equation of motion (leading to formula (13)) of the falling body,  $M \frac{dV}{dt} = - \frac{d}{dt} (mV)$ , no account is taken either of the effect of the weight of the body ( $Mg$ ) or of the resistance of the water during the immersion which may be taken approximately proportional to the square of the velocity  $kV^2$  nor of the effect of the hydrostatic pressure of the water  $\gamma Sh$ . These magnitudes, in particular, the first two, have an effect on the velocity change during impact.

The approximate equation of motion of the body with the above forces taken into account is

$$\frac{d(m + M)V}{dt} = Mg - kV^2 - \gamma Sh \quad (27)$$

where

$$k = \frac{C_D}{2} \rho S$$

$C_D$  resistance coefficient

$S$  wetted area

$\rho$  density of liquid

$h$  depth of immersion

With the aid of equation (27) the results of the tests on the disk are analyzed and also the reasons for the divergence between the experimental and theoretical curve of  $\frac{V}{V_0}$  against  $V_0$ . As is seen from equation (27),

only the weight assists in the fall of the body, the other forces exercising a resistance to the motion. First to be considered is what happens at small initial velocities of impact  $V_0$ . Since the velocity before the start of impact is assumed small, the third term of the equation  $kV^2$  will not have a large value while the weight ( $Mg$ ) does not depend on the velocity and its proportional effect in the equation will be very large. The weight increases the velocity after impact  $V$ ; hence for small initial velocities  $V_0$  the ratio  $V/V_0$  should be greater than the value that would be obtained from the

computation according to the formula  $\frac{V}{V_0} = \frac{1}{1+\mu}$ . The hydrostatic pressure  $\gamma Sh$  is a sufficiently small value and does not appreciably affect the force balance.

The above analysis explains to a considerable extent the experimental results obtained. As has already been pointed out, for small velocities up to  $V_0 \approx 3$  meters per second, the experimental values of  $V/V_0$  increase and become larger than the corresponding theoretical values of  $V/V_0$  (fig. 7). If the velocity before impact  $V_0$  is large, the velocity after impact has a relatively large value and hence also the water resistance  $kV^2$  in the force equation has a considerable value. The weight  $Mg$ , as has been pointed out, does not depend on the velocity, so that its proportional effect in equation (27) decreases as the term  $kV^2$  increases. In other words, with increase in the force at the start of impact there is an increase in the retarding effect of the water and therefore a decrease in the velocity after impact, so that there is a decrease in the ratio  $V/V_0$ . It is also clear from figure 7 that starting with the velocity  $V_0 \approx 3$  meters per second the test curves of  $V/V_0$  against  $V_0$  drop below the corresponding theoretical curves, especially for the weight  $G = 15.66$  kilograms.

The effect of the weight of the body in decreasing  $V/V_0$  may be explained by the fact that with increase in the mass  $M$  the velocity of immersion of the body in the water immediately after impact increases, which in turn brings about an increase in  $kV^2$  — that is, a retarding of the motion and a decrease in the velocity after impact.

If the change in velocity is considered according to the theory of a rigid body, the graph of these velocities as a function of the time would have the appearance of the full curve in figure 8. Actually, however (for large initial velocities  $V_0$ ), by taking account of the term  $kV^2$  the velocity change obtained is that shown by the dotted curve (and the velocity after impact is not equal to  $V$  but to a smaller value  $V_2$ ).

On the basis of the record of the motion of the disk on impact and the high-speed photograph of the fall of the disk on the water it may be concluded that the process of impact does not end at the instant the disk makes contact with the surface of the water as is assumed in the theoretical computation (not taking account of the effect of  $kV^2$  and  $Mg$ ) but somewhat later - that is, after the disk has been immersed in the water to some depth. By the end of the impact process is meant that instant at which the change in velocity becomes very slight. The effect of the water resistance  $kV^2$  on the value of the velocity after impact  $V$  depends on the process of immersion of the disk, the motion of the body being given by equation (27).

To check the effect of the different forces entering the equation of motion on the velocity, the change in the velocity ratio as a function of the initial velocity for various combinations of these forces (the terms of the equation) was computed. (See figs. 9 and 10.)

In computing the velocity after impact  $V_2$  it was assumed that the initial velocity  $V_{01} = \frac{V_0}{1 + \mu}$ . The duration of the impact on the basis of the test curves was chosen as equal to 1.5 times the cut-off interval  $\Delta t$ , since, judging by these curves, the process of rapid change in velocity occurs within one to two intervals  $\Delta t$ .

The change in  $V_2/V_0$  as a function of  $V_0$  was computed for six combinations of the acting forces for  $\Delta t = \text{constant} = 0.016$  second. (See figs. 9 and 10.) From curves I and II (fig. 9) it follows that the hydrostatic forces have a small effect on the value of the ratio  $V/V_0$ . As may be seen from figures 9 and 10, all the curves obtained in taking account of the weight  $Mg$  for small initial impact velocities pass above the straight line  $V_2 = \frac{V_0}{1 + \mu}$ . For large velocities with the resistance  $kV^2$

taken into account all the curves pass below this straight line. The test points for the disk of weight  $G = 8.1$  kilograms lies closest to the curves obtained from the equations

$$M \frac{dV}{dt} = Mg - kV^2 \quad \text{and} \quad M \frac{dV}{dt} = Mg - kV^2 - m \frac{dV}{dt}$$

For the weight  $G = 15.66$  kilograms the test points fall almost exactly on the curve expressed by the equation  $M \frac{dV}{dt} = Mg - kV^2$ . As has been noted, all the curves are constructed for the time interval  $3/2 \Delta t = 0.016$  second. If the computation interval is taken less than 0.016 second, all the theoretical curves approach each other and deviate from the test points.

If it is assumed that the motion of the disk on impact is expressed by the equation  $M \frac{dV}{dt} = Mg - kV^2 - m \frac{dV}{dt}$ , the velocity of immersion of the body as a function of the depth of immersion may be represented in the form

$$V = \frac{1}{c\varphi} \sqrt{\frac{c^2\varphi^2 - 1 + f V_{01}^2}{f}} \quad (28)$$

where

$$f = \frac{0.6\rho Sh}{Mg}$$

$$\varphi = \sqrt{\frac{gfh}{1 + \mu}}$$

$h$  depth of immersion

$V_{01}$  velocity after impact (without taking account of the term  $kV^2$ )

On the other hand, if the velocity of the body  $V$  is known, its depth of immersion can be determined:

$$h = \frac{\sqrt{fV_{01}^2 - 1}}{\sqrt{fV^2 - 1}} \quad (29)$$

Taking the test values of the velocity after impact  $V$  and before impact  $V_0$  and computing  $V_{01}$  by the formula  $V_{01} = \frac{V_0}{1 + \mu}$  yields the corresponding depths of immersion of the body on impact. Thus, for example, there is obtained in the disk tests for  $V_2 = 2.18$  meters per second and  $V_0 = 5.89$  meters per second the value  $V_{01} = 2.53$  and the depth of immersion  $h = 0.046$  meter - that is, the impact ended after the body was immersed 46 millimeters in the water. Measurement on the film of the distance between the last stroke corresponding to the motion of the body in the air and the stroke corresponding to the start of motion of the disk in the water (the impact interval) gave the depth of immersion  $h \approx 40$  millimeters, a result which confirms the above computations.

From the expression for the velocity (obtained from the equation of motion  $M \frac{dV}{dt} = -kV^2 - m \frac{dV}{dt}$

$$V = \frac{MV_{01}(1 + \mu)}{ktV_{01} + M(1 + \mu)} \quad (30)$$

the duration of the impact is found

$$t = \frac{M(1 + \mu)(V_0 - V)}{kVV_{01}} \quad (31)$$

For the initial impact velocity of the disk  $V_0 = 5.89$  meters per second the duration of the impact  $t = 0.019$  second; this time interval is close to that chosen in our computations.

In conclusion, it may be stated that by taking into account the resistance  $kV^2$  and the weight  $Mg$  the theoretical computation of the velocity ratio  $V/V_0$  may be made to agree more closely with the experimental results obtained.

## TESTS ON STRAIGHT V SHAPES (WEDGES)

The tests on wedges were undertaken with several objects in mind. It was first of all desirable to check whether the conclusions drawn from the tests on the disk as regards the necessity for taking into account the resistance force  $kV^2$  and the weight  $Mg$  were applicable to geometric bodies having a shape similar to the steps on the bottoms of seaplanes. A wedge may be considered such a body and may be taken to represent a schematized model of the bottom of a seaplane float. For a body of this shape the theoretical method given by Wagner (reference 5) is available.

It was necessary to explain the limits of applicability of these computations for various values of the initial parameters. In the wedge tests there was also investigated the effect of the V angle on the impact velocities and there was also checked the effect of the  $kV^2$  and the  $Mg$  terms on the velocity ratio  $V/V_0$  for various V angles.

The choice of V angle is of considerable importance in designing the contours of the seaplane, since a proper choice may assure good landing characteristics without disturbing the normal planing conditions. All tests on the wedges were conducted in accordance with the procedure described in section 3 of our present paper. The minimum time interval  $\Delta t$  attained in these tests was equal to 0.00166 second. The wedge dimensions were: length  $l = 1500$  mm; width  $b = 300$  mm; height  $h = 15$  mm; V angle  $\beta = 5^\circ 40'$ . The wedge was constructed of textolite and for greater strength an aluminum sheet of 5-millimeter thickness was attached to the upper surface.

According to the statistical data collected in this investigation, it may be estimated that the magnitude  $\mu = \frac{R}{M}$  in the landing of a full-scale seaplane fluctuates approximately within the limits of 0.3 to 3. Correspondingly the tests on the wedge were conducted for the four values of  $\mu$  as follows:

$$\mu = 3.43; \quad \mu = 2.33; \quad \mu = 1; \quad \mu = 0.468$$

From the same statistical data it follows that the velocities normal to the seaplane bottom in landing fluctuates



within the limits of 2 to 10 meters per second. (These velocities are of fundamental importance in the impact and correspond in our tests to the vertical velocity before impact  $V_0$ .) If it is assumed that the width of bottom selected by us is on the average one-fifth as large as the bottom of the full-scale aircraft, the maximum velocities  $V_0$  for the model tests must be of the order of 4 to 5 meters per second. In our wedge tests at various values of  $\mu$  the maximum initial velocities before impact attained approximately the value of 8 meters per second.

The test results obtained (table 1 and fig. 11) indicate that for all values of  $\mu$  at small velocities  $V_0$  the test curves of  $V/V_0$  against  $V_0$  lie above the theoretical curves (without the terms  $kV^2$  and  $Mg$  taken into account) and conversely for large velocities. With increase in the mass  $M$  of the body (corresponding to a decrease in  $\mu$ ) the deviation between the theory and the tests increases (fig. 12). Starting from a value of  $V_0$  of the order of 2 to 3 meters per second, a slight change in the ratio  $V/V_0$  with further increase in the initial velocity  $V_0$  may be noted. This confirms what was stated in section 2 with regard to the theoretical considerations.

TABLE 1

Weight $G$ (kg)	Associated mass $m = \frac{\pi}{8} \rho B^2 l \xi_1 \xi_2$ (kg sec <sup>2</sup> /m)	$\mu = \frac{m}{M}$	$\frac{V}{V_0} = \frac{1}{1 + \mu}$	Test value, $\frac{V}{V_0}^{**}$
13.6	4.76	3.43	0.226	0.22
20	4.76	2.33	.3	.27
47	4.76	1.0	.5	.418
97.6	4.76	.468	.684	.59

\* $M = \frac{G}{g}$  mass of wedge.

\*\*Values of  $V/V_0$  for study portion of curve  $\frac{V}{V_0} = f(V_0)$ .

The values of  $\mu$  most often occurring in the landing of a seaplane fluctuate from 0.3 to 1.5 and it is just for these values of  $\mu$ , according to the curve of figure 12, that the deviation between the experimental and theoretical values of  $V/V_0$  is greatest (of the order of 15 to 20 per cent). (A certain effect on the test values of  $V/V_0$  is produced by the elasticity of the tested body.)

The problem to what extent the theoretical computation may be made to approach more closely the test results by taking account of the water resistance  $kV^2$  and the weight of the body  $Mg$  in the equation of motion now will be considered. If it is assumed that from the start of the impact the motion of the wedge satisfies the equation for the immersion of the body

$$M \frac{dV}{dt} = Mg - kV^2 - m \frac{dV}{dt}$$

(for the initial conditions  $t = 0$ ;  $V = V_0$ , the velocity before impact). Then, having constructed the curves of  $V$  against  $t$  and knowing, even approximately, the duration of the impact, it is possible to determine the additional drop in velocity during impact as a result of the action of the forces  $Mg$  and  $kV^2$ . On figure 13 curves of  $V$  against  $t$  are drawn for the initial velocity of impact (immersion)  $V_0 = 5$  meters per second and various weights of the wedge  $G$  and values of  $\mu$ . With the aid of these curves it is possible to compute the velocity ratio during impact with the additional drop in velocity taken into account.

As an example, let us assume an initial velocity before impact  $V_0 = 5$  meters per second (for a wedge of weight  $G = 20$  kg). The theoretical value of the ratio

for the given case is  $\frac{V}{V_0} = 0.3$  (without the corrections)

and the velocity after impact (theoretical) is then  $V = 5 \times 0.3 = 1.5$  meters per second. Correction to this value will be made, account being taken of the additional drop in velocity. Since the duration of the impact computed by the formula

$$t = \frac{2h_2 \left( 1 + \frac{gm}{3p} \right)}{\pi V_0}$$

for the example given is equal approximately to 0.0041 second, the drop in velocity during immersion must be taken for the same time interval  $\Delta t$  which from the graphs gives  $\Delta V \approx 0.4$  meter per second. The velocity after impact is therefore  $V_2 = V - \Delta V = 1.5 - 0.4 = 1.1$  meters per second, and the velocity ratio  $V_2/V_0 = 0.24$ .

As may be seen from the above example by taking into account the terms  $kV^2$  and  $Mg$ , the theoretical values of  $V/V_0$  are made to approach more closely the experimental values. It should be pointed out that the assumed correction for the additional drop in velocity is somewhat exaggerated since, in the computation, it was assumed that the wetted area of the wedge during the entire time of immersion remained constant and equal to the maximum value  $S$ ; and this led to a decrease in the computed ratio  $V/V_0$ . It should also be noted on the basis of the film record analysis that actually the duration of the impact of the wedge is greater than follows from the computation. (Thus, for example, for the velocity  $V_0 = 5$  meters per second the computed value of  $\Delta t$  is 0.0041 second and the experimental 0.005 second.) This fact serves as additional confirmation of the effect of the resistance on the impact.

To investigate the effect of the  $V$  angle on the impact velocities, three wedges were tested with the following data: length  $l = 500$  mm; width  $B = 100$  mm; and  $V$  angle  $\beta = 5^\circ 40'$ ,  $23^\circ$ , and  $30^\circ$ . (These dimensions correspond to 1/3 scale reduction.) The test results (fig. 14) show that with increase in the  $V$  angle there is an increase in the velocity ratio  $V/V_0$ , a fact which is explained by the decrease in the associated mass for greater  $V$  angles, as also follows from theoretical considerations. (See reference 4.)

The test data and the order of magnitude of the deviation between the computed and test results are given in table 2.

TABLE 2

Wedge	$V$ angle	Weight, $G$ (kg)	$\mu$	$\frac{V}{V_0} = \frac{1}{1+\mu}$	$\frac{V}{V_0}$ (experimental)
2	$5^\circ 40'$	3.72	0.468	0.684	0.585
3	$23^\circ$	3.72	.415	.707	.66
4	$30^\circ$	3.72	.396	.718	.705

The general character of the three curves of  $V/V_0$  against  $V_0$  is analogous to the curve obtained in testing wedge 1 and the disk. A considerable deviation of the test curve from the theoretical is obtained for  $\beta = 5^\circ 40'$ , a fact which may be explained by the large effect of the water resistance  $kV^2$  for bodies of small  $V$  angle since the values of  $C_D$  and  $S$  entering the coefficient  $k$  for the case under consideration are large. It may be generally concluded, however, on the basis of figure 14, that for large angles (at given  $\mu$ ), the effect of the weight  $Mg$  and the resistance of the water  $kV^2$  is not large. Thus, in summary, it may be said that the results obtained previously on the disk tests are confirmed for the wedge tests.

#### IMPACT ON WATER OF A SLOTTED WEDGE AND DISK WITH APERTURE

The associated mass coefficient of two plates in tandem (reference 22), (fig. 15) is less than the associated mass of a single plate having a width equal to the sum of the widths of both plates. In other words, a small slot between the plates considerably decreases the coefficient of associated mass as compared with the value for one continuous plate (with the relative slot width  $p^* = 0$ ).

The associated mass coefficient for two flat plates in tandem is expressed by the formula

$$\lambda_y = \pi\rho \left[ \frac{1}{4}(b_2 - b_1 + a_2 - a_1)^2 - \frac{E(k)}{F(k)} (a_2 - a_1)(b_2 - b_1) \right] \quad (32)$$

( $a_1, b_1, a_2, b_2$  are the coordinates of the ends of the plate (fig. 16)) where  $E(k)$  and  $F(k)$  are the complete elliptic integrals of the first and the second order and

$$k^2 = \frac{(b_2 - a_2)(b_1 - a_1)}{(a_2 - a_1)(b_2 - b_1)}$$

Setting

$$\frac{a_2 - b_1}{b_1 - a_1} = p^*; \quad \frac{b_2 - a_2}{b_1 - a_1} = q$$

gives

$$k = \frac{q}{(1 + p^*)(p + q)}$$

where

$p^*$  ratio of width of slot to width of plate

$q$  ratio of widths of plates

For the associated mass coefficient there is obtained

$$\lambda_y = \mu^*(p_1^*q) \frac{\rho\pi}{4} [(b_1 - a_1)^2 + (b_2 - a_2)^2] \quad (33)$$

where

$$\mu^*(p_1^*q) = \frac{(1 + 2p + q)^2 - 4 \frac{E(k)}{F(k)} (1 + p)(p + q)}{1 + q^2} \quad (34)$$

The factor  $\mu^*$  takes into account the mutual action of the plates. For  $p^* = 0$  and  $q = 1$ , the coefficient  $\mu^*$  has a maximum value equal to 2. Figure 16 gives curves of  $\mu^*$  against  $p^*$  for constant values of  $q$ .

The value of the associated mass for three plates in tandem was computed by V. Shupanov (reference 22). (The formulas for the associated mass are given in the special case where the plates are symmetrical about the center line of the middle plate (fig. 17).) The associated mass per unit length of the three plates is given by the formula

$$m = \rho\pi \left[ \frac{1}{2} (c^2 + b^2 - a^2) - (c^2 - a^2) \frac{E\left(\frac{\pi}{2}, k\right)}{F\left(\frac{\pi}{2}, k\right)} \right] \quad (35)$$

where

$$k^2 = \frac{c^2 - b^2}{c^2 - a^2}$$

By introduction of the magnitude  $v^*$ , a nondimensional coefficient expressing the mutual effect of the plates on one another, there is obtained the following expression for the associated mass  $m$ :

$$m = v^* \frac{\rho \pi}{2} \left[ a^2 + \frac{(c-b)^2}{2} \right] \quad (36)$$

For the case of slots between the plates the coefficient  $v^*$ , and hence also the associated mass  $m$ , sharply decrease. Figure 18 shows the change in the coefficient

$v^*$  as a function of  $\frac{c-b}{b}$  and  $\frac{a}{b}$ . For  $\frac{a}{b} = 1$  - that is, in the absence of slots -  $v^*$  is equal to 3; for  $\frac{a}{b} = 0$  and  $\frac{c-b}{b} \ll 1$  - that is, for two widely separated plates -  $v^* \approx 1$ .

The results of the drop tests of a slotted wedge in water are presented below. The slotted wedge was made up of three plates separated 15 millimeters from one another. The over-all width of the slots constituted 10 percent of the width of the wedge. The dimensions of the wedge were the following (figs. 19 and 20): length  $L = 1.5$  m; over-all width  $b = 0.3$  m; outer V angle  $\beta = 50^\circ 40'$ ; width of each part of wedge 90 mm; height of wedge  $h = 0.015$  m. The individual plates of the wedge (made of beech) were connected by means of an aluminum plate of 5 millimeters thickness. Over the slots in the metal plate round apertures of 15 millimeters diameter were cut out, spaced 5 millimeters from each other. In its over-all dimensions the slotted wedge corresponded to the previously tested continuous wedge, a fact which, taken together with the identical test weights of both wedges made it possible to estimate the effect of the slots on the impact velocities.

With the aid of formula (35) the associated mass of the slotted wedge was computed and in the value obtained corrections were made for the finiteness of the span and the V angle as in the case of the continuous wedge. The associated mass per unit length of the three plates is

$$\frac{m}{l} = \rho \pi \frac{1}{2} (c^2 + b^2 - a^2) - (c^2 - a^2) \frac{E\left(\frac{\pi}{2}, k\right)}{F\left(\frac{\pi}{2}, k\right)}$$

$$k^2 = 0.923; \quad k = \sin \varphi = 0.9607; \quad \varphi = 73^\circ 44';$$

$$E\left(\frac{\pi}{2}, k\right) = 1.08662; \quad F\left(\frac{\pi}{2}, k\right) = 2.69314; \quad \frac{m}{l} = 1.2734$$

whence

$$m = 1.91$$

since

$$l = 1.5$$

The corrected associated mass is

$$m_{\xi} = m \xi_1 \xi_2$$

where

$$\xi_1 = 0.98 \text{ the correction for the } V \text{ angle}$$

$$\xi_2 = 0.903 \text{ the correction for finite span}$$

$$m_{\xi} = 1.69$$

The test on the slotted wedge was conducted for two weights  $G_1 = 12.52$  and  $G_2 = 13.6$  kilograms. For these weights, according to the computed value of the associated mass, the velocity ratio is given by the following figures

$$G_1 = 13.6 \text{ kg}; \quad G_2 = 12.52 \text{ kg};$$

$$\mu = \frac{m_{\xi}}{G} = 1.22; \quad \mu = \frac{m_{\xi}}{G} = 1.309$$

$$V = \frac{V_0}{1+\mu} = \frac{V_0}{2.201} = 0.450 V_0; \quad V = \frac{V_0}{1+\mu} = \frac{V_0}{2.309} = 0.433 V_0$$

For the continuous wedge for the same loads the corresponding computation gives

$$V = 0.226 V_0 \text{ (for } G = 13.6 \text{ kg)}$$

and

$$V = 0.212 V_0 \text{ (for } G = 12.52 \text{ kg)}$$

From the above results it is seen that the velocity ratio for the case of the slotted wedge (with the slot dimensions given above) is about twice as large as for the continuous wedge.

The test procedure for the two wedges was identical. The minimum time intervals  $\Delta t$  fixed on the photographic film was equal to 0.0111 second for  $G = 12.52$  kilograms and 0.016 second for  $G = 13.6$  kilogram.

The test results (fig. 21) showed that the velocity ratio  $V/V_0$  for the slotted wedge was not actually twice as large as for the continuous wedge, as was obtained from the computation. At large initial impact velocities ( $V_0$  from 3.6 to 6 m/sec) the value of the velocity ratio for the slotted wedge was greater than that for the continuous wedge by approximately 10 percent, while at small initial velocities it was greater by 50 percent.

The obtained deviation between the computed velocity values (according to Wagner) for the slotted wedge and the test results may be explained evidently by the fact that with the method used no account was taken of the resistance of the water on immersion of the body which to a large extent depends on the velocity.

As all our previous impact tests have shown the velocities before and after impact are not determined by

the ratio  $\frac{V}{V_0} = \frac{1}{1 + \mu}$  (according to Wagner). In other

words, although the associated mass for the slotted wedge is half as large as that for the continuous wedge (according to the theory), the velocity ratio does not become twice as large because with decreased associated mass there is an increase in the velocity of immersion and hence an increase in the resistance of the water  $kV^2$  - that is, an additional decrease in the velocity after impact and the ratio  $V/V_0$ . In fact, even a rough correction for the effect of the water resistance improves the agreement between theory and experiment. As an example, consider the case of impact of a slotted wedge (of weight  $G = 13.6$  kg) with initial velocity  $V_0 = 5$  meters per second. The velocity ratio, according to

the formula  $\frac{V}{V_0} = \frac{1}{1 + \mu}$  for the given case is equal to

0.454; hence the velocity after impact is equal to  $V = 0.454 \times 5 = 2.27$  meters per second. The correction for the water resistance after an interval  $\Delta t = 0.005$  second is, according to the curve (fig. 13,  $\mu = 1.203$ ), equal to  $\Delta V = 0.95$  meter per second. By application



of this correction a new value is obtained for the velocity after impact  $V_2 = V - \Delta V = 2.27 - 0.95 = 1.32$  meters per second and for the velocity ratio  $V_2/V_0 = 0.264$ . According to the test value of  $V/V_0$  for  $V_0 = 5$  meters per second is 0.227. Notwithstanding the rough assumptions made, it is evident from the example given that the applied corrections bring the theoretical computations in closer agreement with the test results.

For additional checking of the results obtained with the slotted wedge special tests were conducted to determine the associated mass of the wedge. The direct determination of the associated mass is possible with the aid of the Pabst method by which the associated mass can be found from the oscillation period of the body in the water and in air. (See references 20 and 21.)

The test setup (fig. 22) was first checked by testing on it the continuous wedge for which an exact formula of the associated mass is available ( $m = \frac{\pi}{8} \rho B^2 l$ ). The tested plate was of aluminum of length  $l = 250$  mm; width  $B = 50$  mm; weight  $G = 126.8$  g; and thickness  $h = 3$  mm. The edges of the plate were rounded.

A comparison of the computed with the test results gives the following figures:  $m_{\text{test}} = 0.045$  and  $m_{\text{comp}} = 0.0454$  (in the computed value a correction was applied for the finite span). The results indicate good operation of the test setup.

The value of the associated mass of the slotted plates was checked for three models. (See fig. 23.) All plates were of the same length  $l = 250$  millimeters and the same over-all width  $B = 50$  millimeters. (By over-all width is meant the total width of two component plates plus slot.) One of the plates had a slot of width 4 millimeters (10 percent of the total area), another 16.5 millimeters (33.3 percent), and the third 1.2 millimeters (2.4 percent).

The results of the tests with the slotted plates (table 3) gave very good agreement with the test values of the associated mass obtained by the formulas of L. Sedov ( $m = \lambda_y l$ ).

TABLE 3

## TEST RESULTS FOR DETERMINATION OF ASSOCIATED MASS OF PLATES

Plate	Width of slot (mm)	Weight of plate (g)	Associated mass, m $\left(\frac{\text{kg} \cdot \text{sec}^2}{\text{m}}\right)$	Computed associated mass, m $\left(\frac{\text{kg} \cdot \text{sec}^2}{\text{m}}\right)$
1	0	126.75	0.045	0.0454
2	1,2	89.2	.0254	.0276
3	5	112.5	.0235	.0211
4	16.5	76.62	.01095	.01063

Thus the tests on the slotted wedges entirely confirmed both the theoretical prediction of a decrease of the associated mass in the presence of a slot and the assumption that the decrease in associated mass does not strongly affect the impact velocity ratio caused by the additional effect of the water resistance. It is thus seen that the computation of the impact by the Wagner theory does not correspond to the true conditions in a number of cases.

In addition to the slotted wedge tests there were also tested a solid disk of diameter  $D = 120$  millimeters and the same diameter disk with round aperture diameter  $d = 26.4$  millimeters (the area of the aperture being 5 percent of the total disk area). The obtained experimental value of the associated mass for the solid disk almost agreed with the computed value ( $m = 4/3 \rho r^3$ ), while for the disk with aperture, since no theoretical value for its associated mass is available, a corresponding comparison with experimental results could not be made. Judging, however, by the results of the experiments, it may be said (table 4) that the presence of an aperture of 5 percent of the disk area decreased the associated mass by 23 percent.

TABLE 4

Object tested	Weight (m/g)	Test value of associated mass $\left(\frac{\text{kg sec}^2}{\text{m}}\right)$	Theoretical value of associated mass $\left(\frac{\text{kg sec}^2}{\text{m}}\right)$
Disk . . . . .	87	0.0580	0.0586
Disk with aperture . . .	83.5	.0449	

A similar disk (total diameter  $D = 500$  mm; diameter of aperture  $d = 110$  mm) also was tested by the drop method described. (See section 3.) The results of these tests did not show any considerable changes in the velocities before and after impact as compared with the solid disk described in section 4. (See fig. 24.)

## CONCLUSION

On the basis of the results obtained, the following conclusions may be drawn:

1. The landing on water of a freely falling body is determined by two nondimensional coefficients: namely, the Froude number and the mass coefficient. For the case of a seaplane landing on water the impact phenomenon is determined by only the mass coefficient.

2. The results obtained in computing the impact by the method of Wagner give a disagreement between the computed and the test results in a number of cases. The added correction in the equation of motion for the weight of the body  $Mg$ , the hydrostatic forces  $\gamma Sh$ , and the resistance of the water  $kV^2$  considerably improves the agreement of the theory with the test results.

3. An increase in the angle of a V shape (within the range of  $5^\circ$  to  $30^\circ$ ) considerably decreases the velocity change on impact.

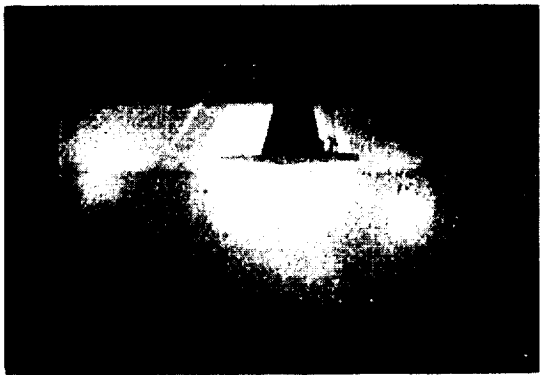
Translation by S. Reiss,  
National Advisory Committee  
for Aeronautics.

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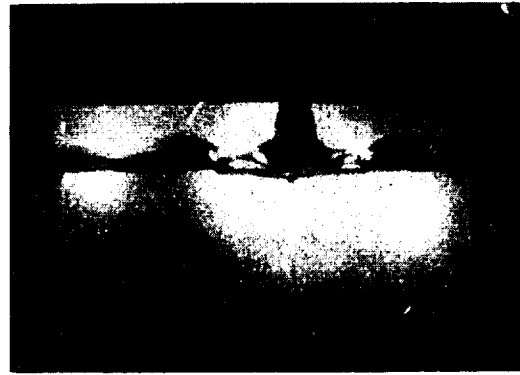
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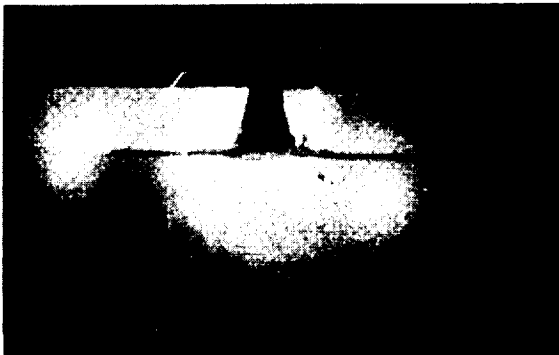




$t = 0$



$t = 0.016 \text{ sec}$



$t = 0.006 \text{ sec}$



$t = 0.176 \text{ sec}$

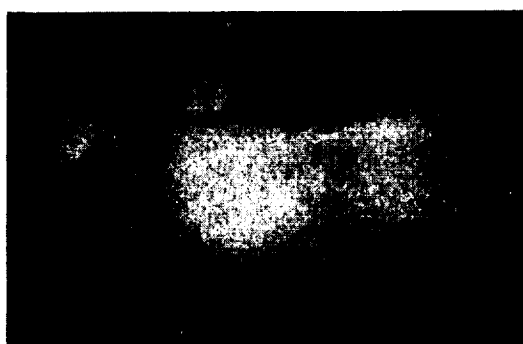


$t = 0.288 \text{ sec}$

Figure 1.- Pictures of drop of wedge on water.  
Height of drop,  $H = 1 \text{ m.}$



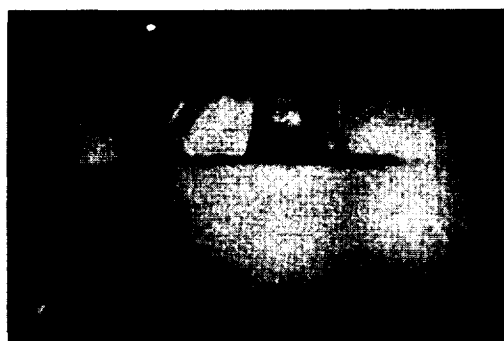




$t = 0$



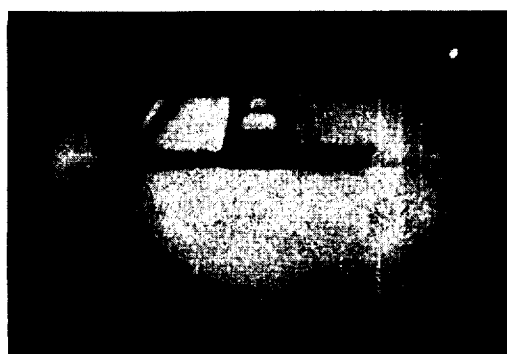
$t = 0.0119 \text{ sec}$



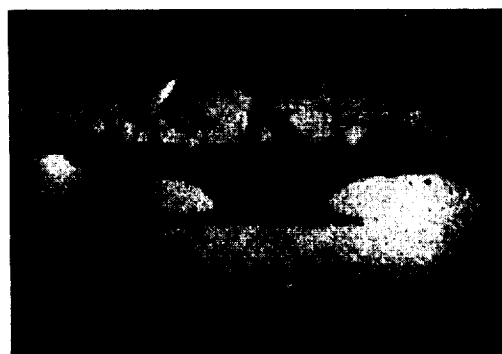
$t = 0.0068 \text{ sec}$



$t = 0.1173 \text{ sec}$



$t = 0.0085 \text{ sec}$



$t = 0.2975 \text{ sec}$

Figure 2.- Pictures of drop of disk on water.  
Height of drop,  $H = 1 \text{ m.}$



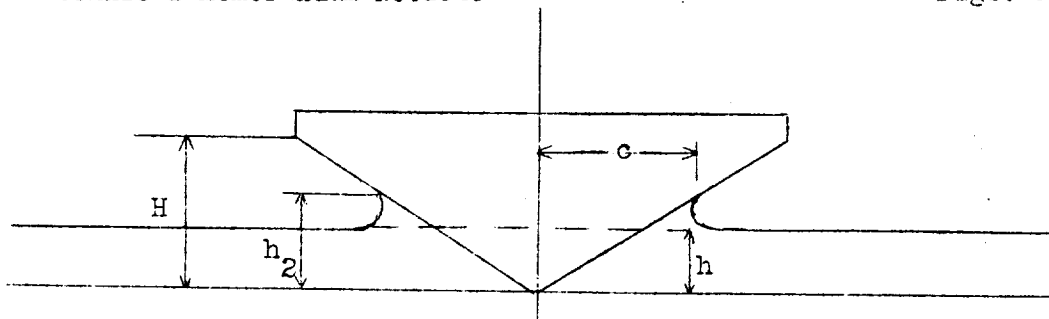


Figure 3

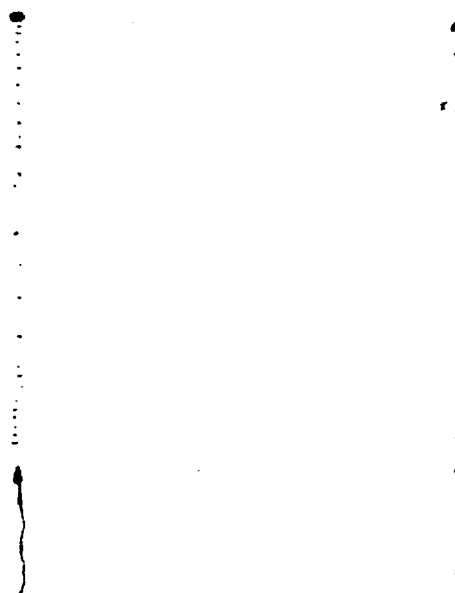


Figure 5.- Record of motion of body falling on water.

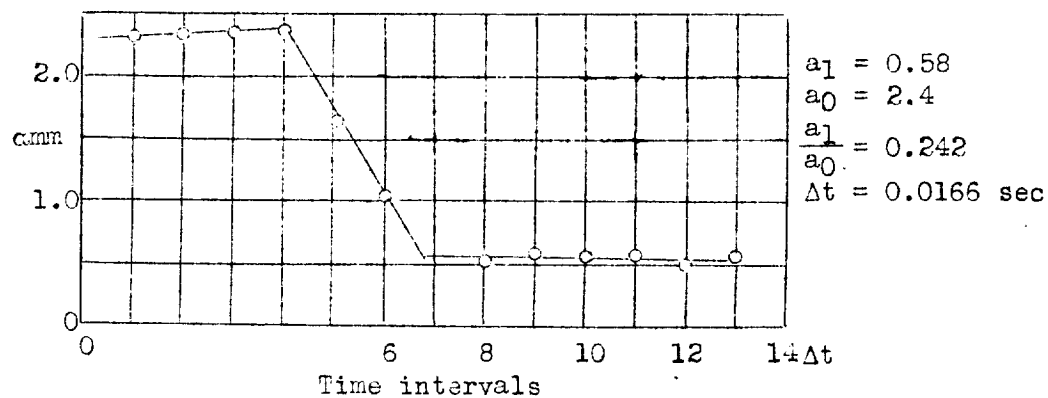


Figure 6.- Typical diagram of distance traversed by body in falling on water (obtained from film readings).





Figure 4.- General view of wedge tested.

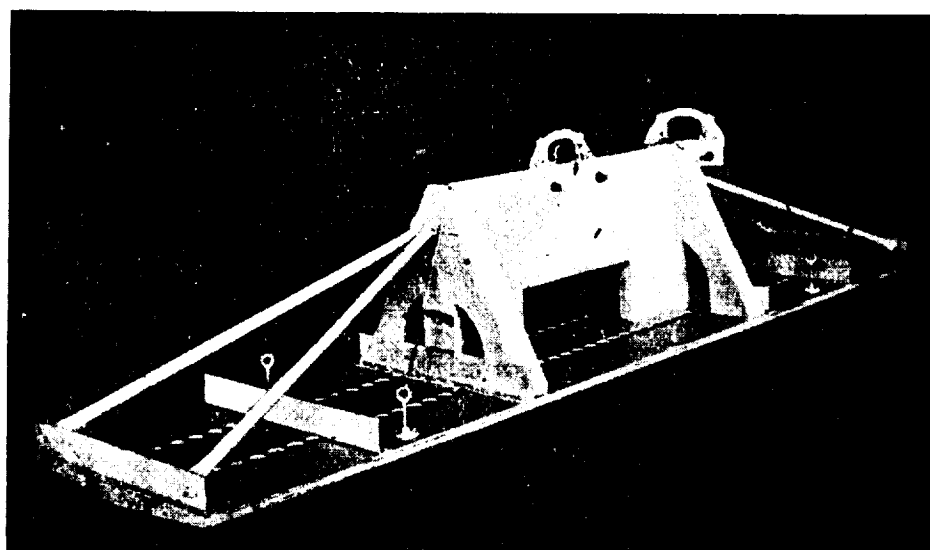


Figure 19.- General view of slotted wedge tested.

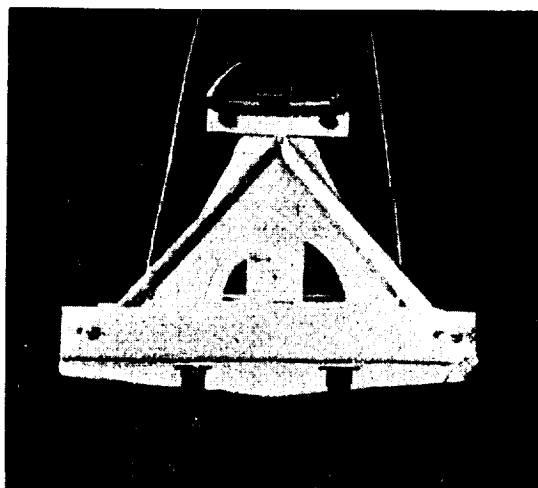


Figure 20



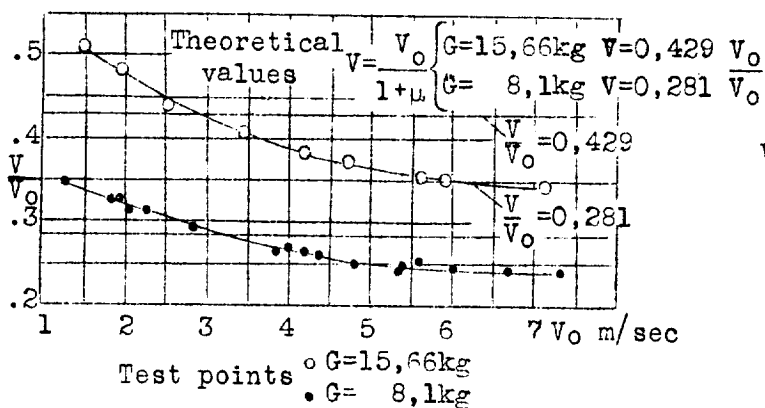


Figure 7.- Experimental and theoretical curves of ratio  $V/V_0$  against  $V_0$  obtained in tests on disks of various weights.

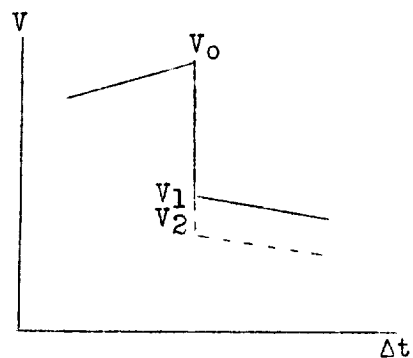


Figure 8.- Curve of change of velocity of drop  $[V=f(t)]$  with and without the resistance of the water ( $kV^2$ ) taken into account.

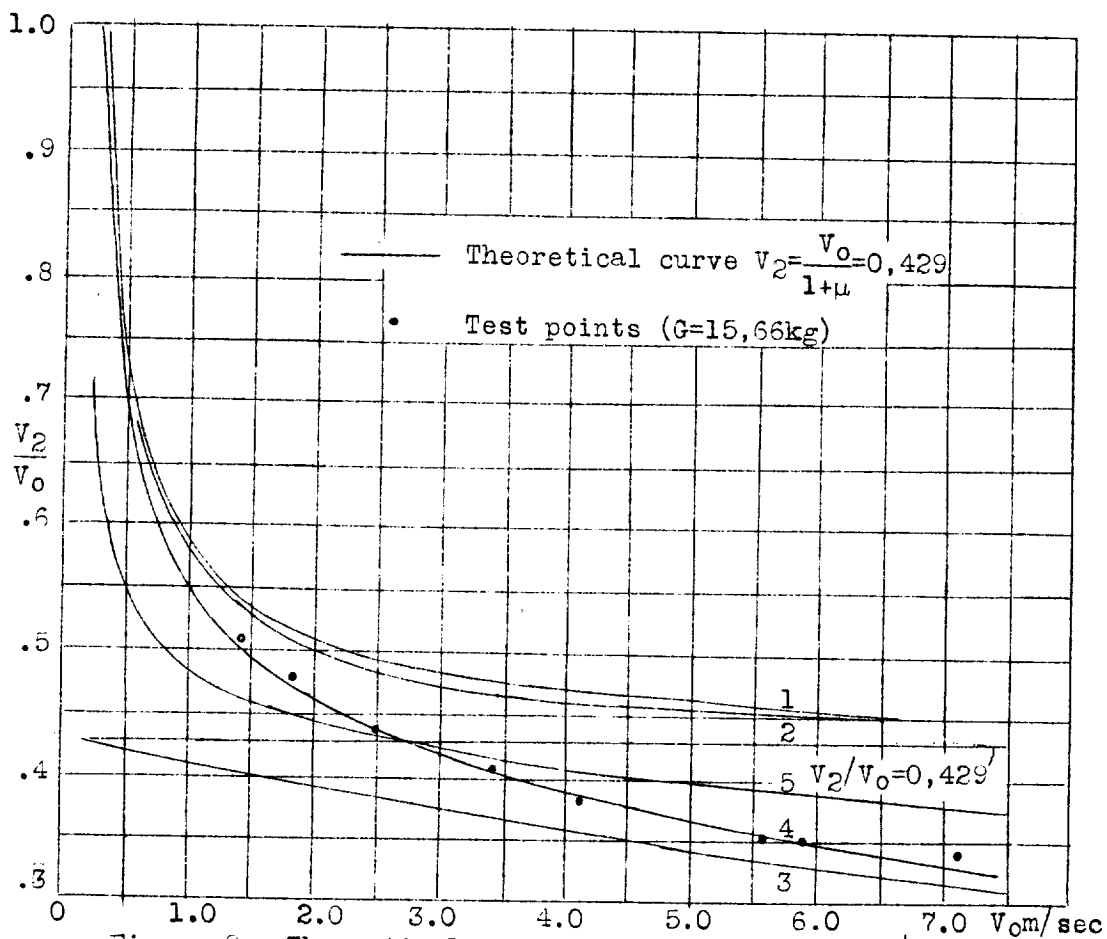


Figure 9.- Theoretical curves of velocity ratio  $V/V_0$  against  $V_0$  (for the time interval  $\Delta t = 0.016$  sec) with account taken of the various forces acting on an immersed disk of weight  $G = 15.66$  kg.





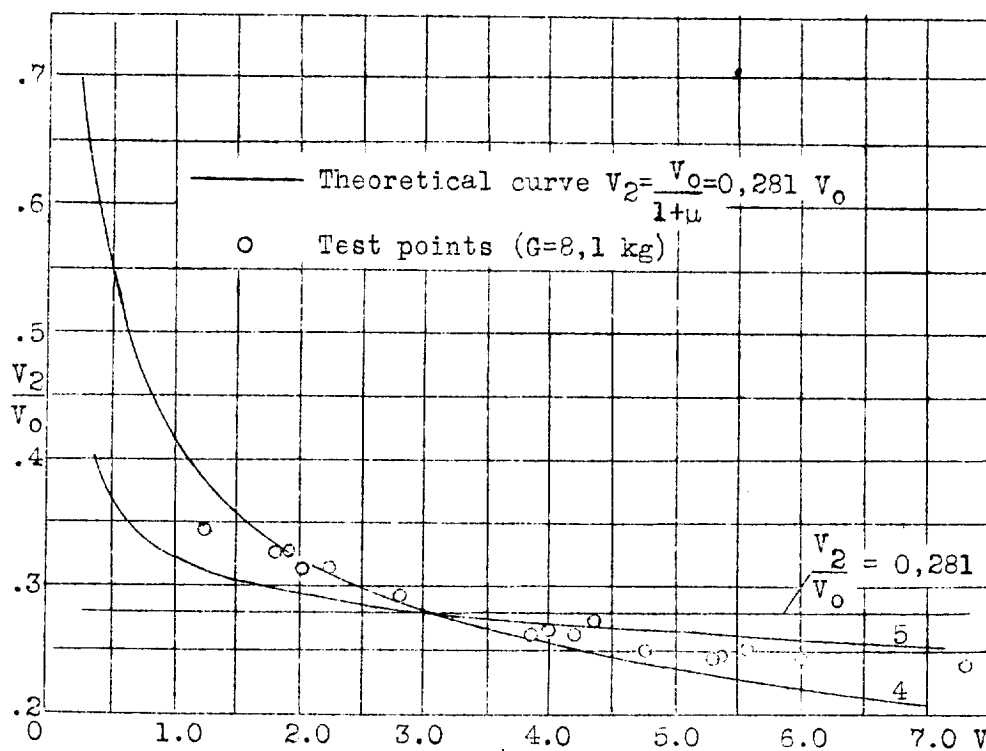


Figure 10.- Theoretical curves of  $V/V_0$  against  $V_0$  (for time interval  $\Delta t = 0.016 \text{ sec}$ ) with account taken of the various forces acting on an immersed disk of weight  $G = 8.1 \text{ kg}$ .

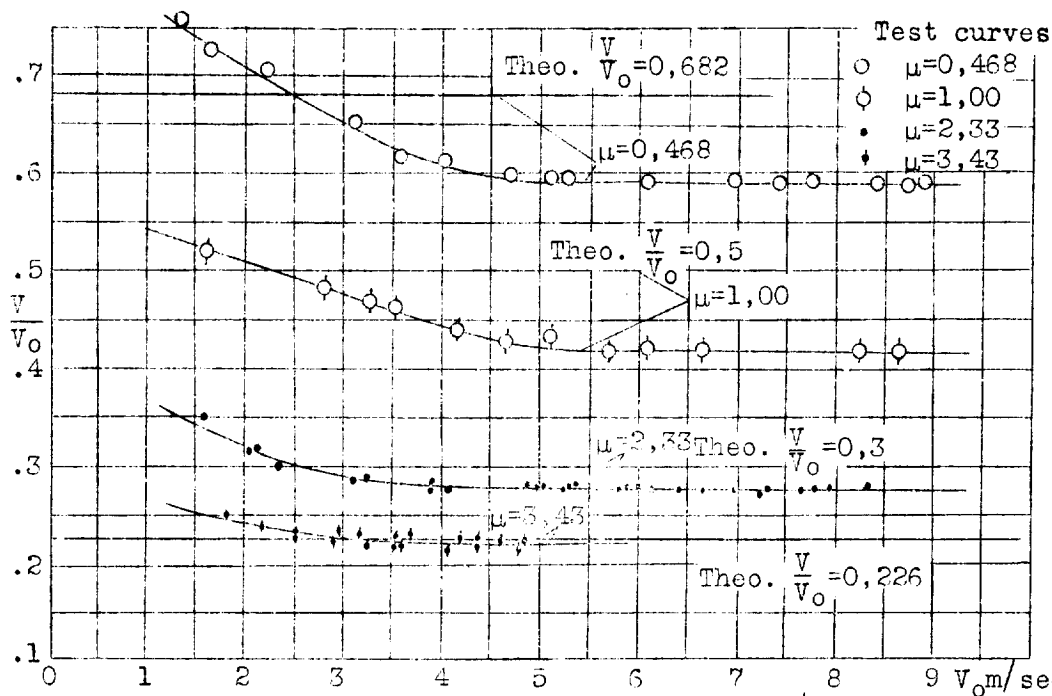


Figure 11.- Experimental and theoretical curves of  $V/V_0$  against  $V_0$  obtained in testing a wedge for various values of  $\mu$ .



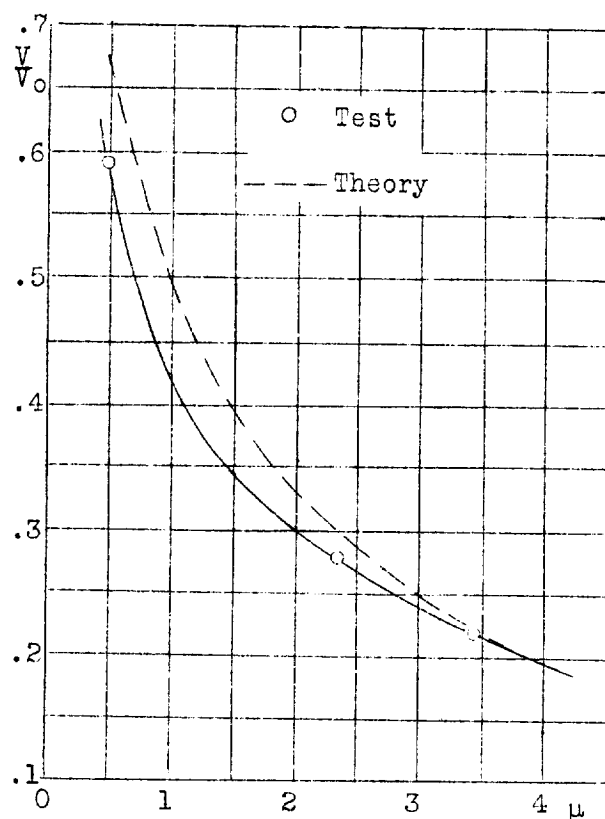


Figure 12.- Experimental and theoretical curves of  $V/V_0$  against  $\mu$  for a wedge of angle  $\beta = 50^\circ 40'$ .

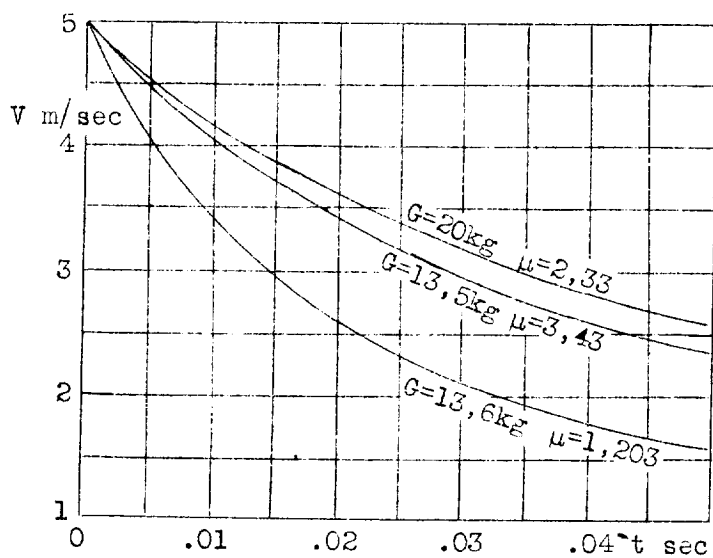


Figure 13.- Curves of velocities of immersion of a wedge  $[V=f(t)]$  with account taken of the resistance  $kV^2$  and the weight  $Mg$ .



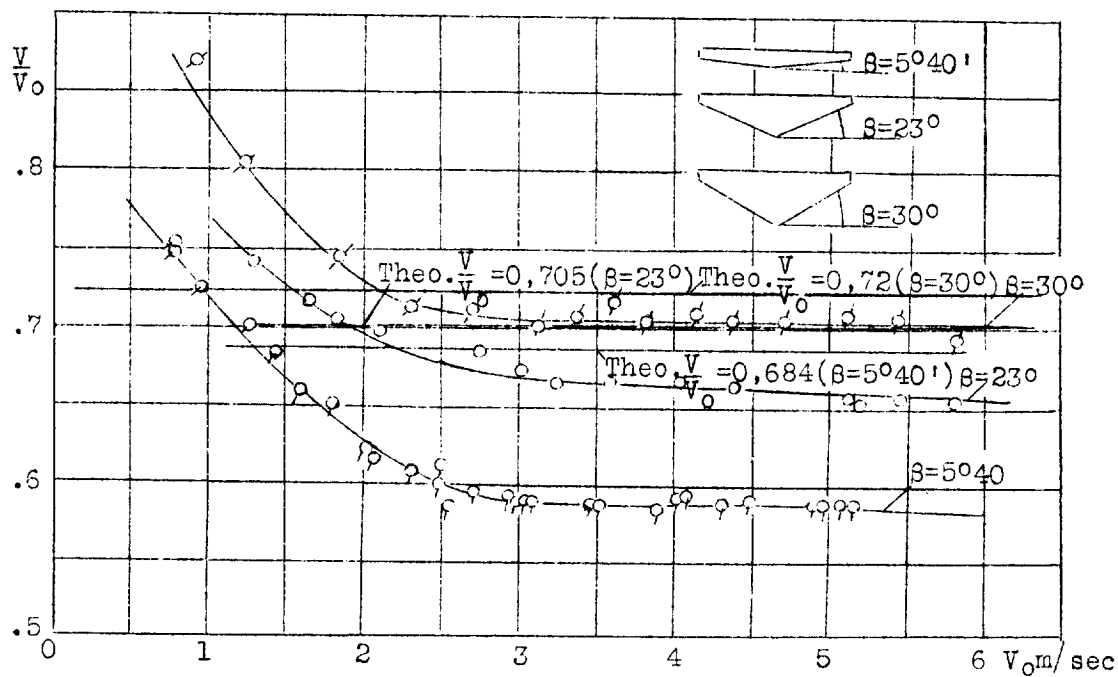


Figure 14.- Experimental and theoretical curves of  $V/V_0$  against  $V_0$  for wedges of various angles.

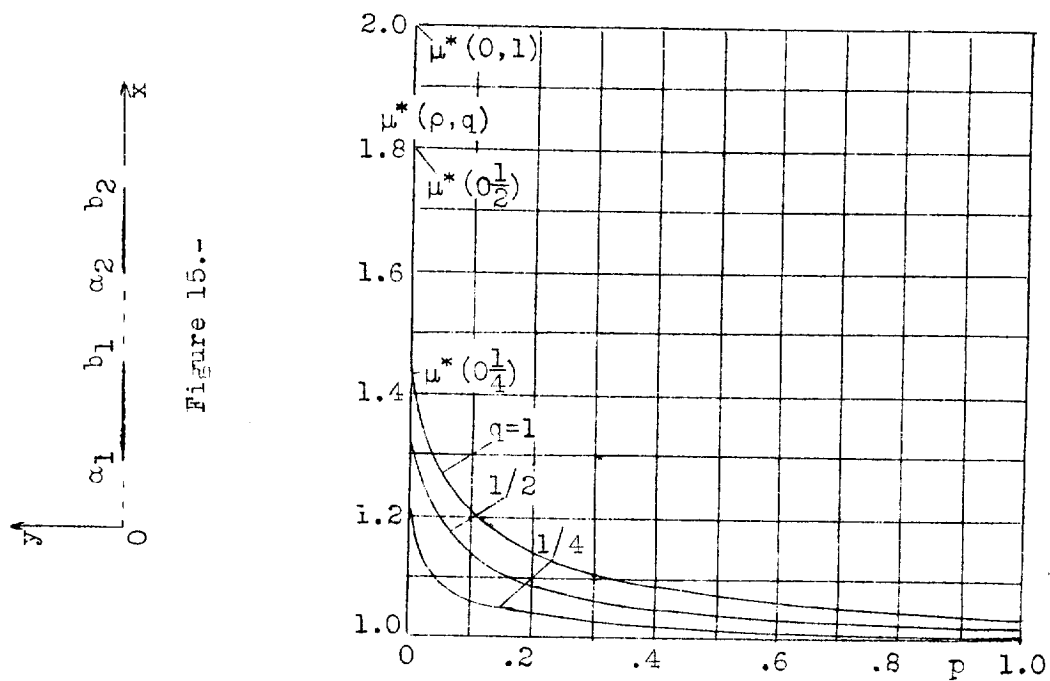


Figure 16.- Curves of  $\mu^*$  against relative slot width  $p^*$ .



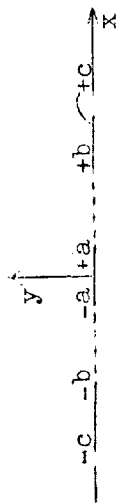


Figure 17.-

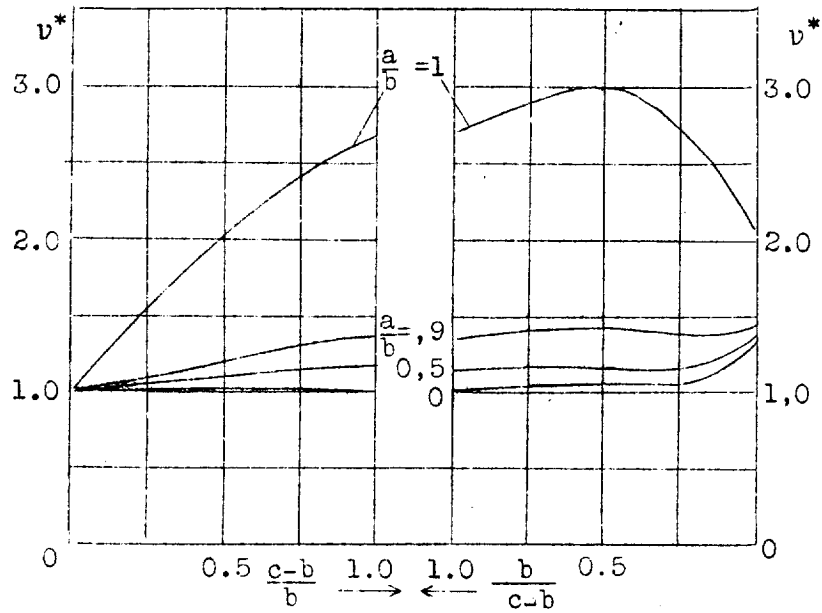


Figure 18.- Curve of  $v^*$  against  $c-b/b$  for various values of  $a/b$ .

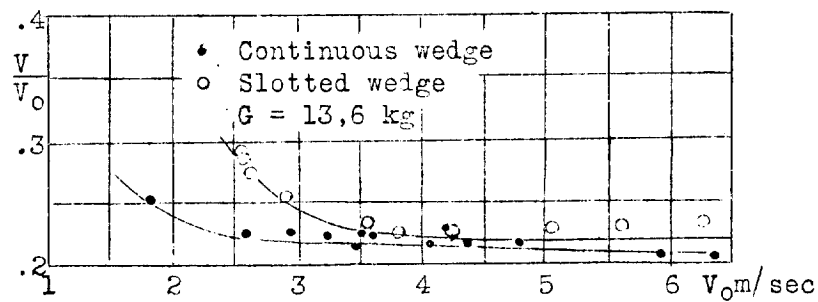


Figure 21.- Experimental curves of  $V/V_0$  against  $V_0$  obtained for continuous and slotted wedges.

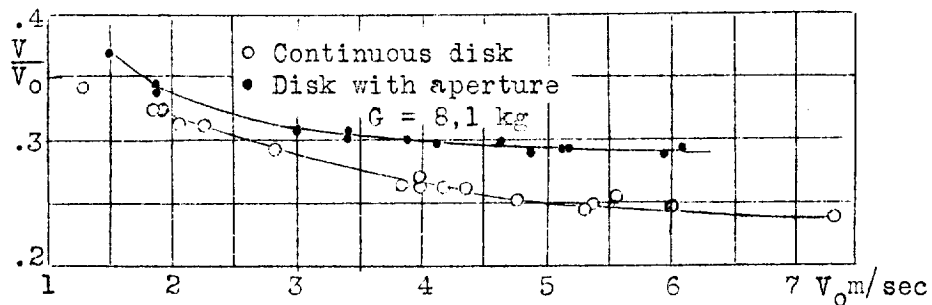


Figure 24.- Experimental curves of  $V/V_0$  against  $V_0$  obtained for solid disk and disk with aperture.





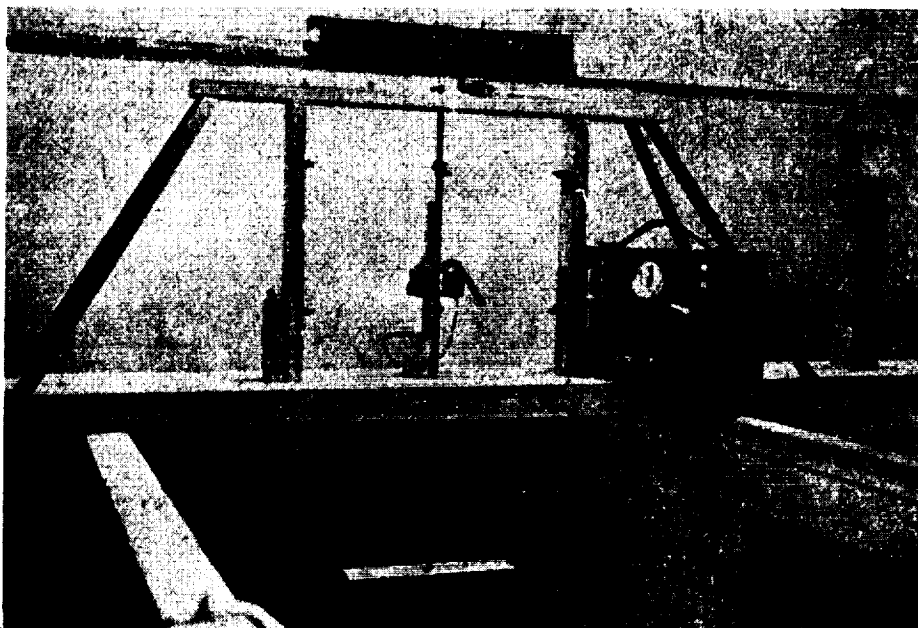


Figure 22.- Set up for determination of associated mass.



Figure 23.- Plates tested.



